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Journal of Sound and Vibration (1980) 72(4), 547–549

(11) 2 June 80 (12) 3

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COMMENTS ON "VIBRATION OF A ROTATING BEAM WITH TIP MASS"

Recently Hoa [1] presented a finite element model to obtain the vibration frequencies of a rotating beam with tip mass. It was discovered later that there was a minus sign missing in the expression for d in Table 1 of reference [1]†; the correct expression should be $d = -(R' + n)$. The correct expression for d gives faster convergence in the finite element results and slightly smaller values than those reported in reference [1]. A revised version of Table 3 of reference [1] is shown in Table 1. In Table 1, a value of $\lambda = 10{\cdot}70310$ is given instead of the value of $\Omega = 500$ rad/s in Table 3 of reference [1] based upon the physical parameters of $h = 0{\cdot}02$ in, $L = 5$ in, $E = 30 \times 10^6$ lb/in^2, $\rho = 0{\cdot}283$ lb/in^3, and $g = 386$ in/s^2.

TABLE 1

Convergence of the finite element model, $\lambda = 10{\cdot}70310$, $R = \gamma = 0$

Mode	Vibration frequency parameter $\bar{a}$: 10 elements	12 elements	14 elements
First	11·8926	11·8924	11·8924
Second	35·0044	35·0037	35·0034
Third	76·3471	76·3394	76·3360
Fourth	136·889	136·835	136·811
Fifth	216·963	216·726	216·621
Sixth	317·204	316·446	316·101

TABLE 2

Comparison between the present finite element results and calculations based on [2], first mode, $R = \gamma = 0$

λ	$\bar{a}$: Present, 10 elements	$M = 10$, $N = 3$ [2]	$M = 1$, $N = 15$ [2] (exact)
0	3·51602	3·51602	3·51602
1	3·68165	3·68165	3·68165
2	4·13732	4·13732	4·13732
3	4·79728	4·79728	4·79728
4	5·58501	5·58501	5·58500
5	6·44957	6·44957	6·44954
6	7·36041	7·36041	7·36037
7	8·29970	8·29970	8·29964
8	9·25694	9·25694	9·25684
9	10·2258	10·2258	10·2257
10	11·2026	11·2026	11·2023
11	12·1847	12·1847	12·1843
12	13·1706	13·1706	13·1702

† The results of reference [1] were first noted to be incorrect by Dr Dewey H. Hodges. The actual sign error was later discovered by Dr Michael J. Rutkowski.

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For the case of a rotating beam without a tip mass ($\gamma = 0$), the present finite element results obtained by using a third order polynomial and ten elements are presented for various values of the angular velocity parameter λ. Results obtained by using Hodges' analysis [2] with both the present element configuration ($M = 10$, $N = 3$) and with the fifteenth order polynomial and one element ($M = 1$, $N = 15$), which yields the essentially exact solution, are also shown. For ten elements, the results obtained from the present analysis and that of reference [2] are identical and agree quite well with the exact solution.

TABLE 3

Comparison between the present finite element results and calculations based on [2], *second mode*, $R = \gamma = 0$

	$\bar{\alpha}$		
λ	Present, 10 elements	$M = 10$, $N = 3$[2]	$M = 1$, $N = 15$ [2] (exact)
0	22·0352	22·0352	22·0345
1	22·1817	22·1817	22·1810
2	22·6156	22·6156	22·6149
3	23·3210	23·3210	23·3203
4	24·2740	24·2740	24·2733
5	25·4468	25·4468	25·4461
6	26·8098	26·8098	26·8091
7	28·3349	28·3349	28·3341
8	29·9963	29·9963	29·9954
9	31·7716	31·7716	31·7705
10	33·6416	33·6416	33·6404
11	35·5905	35·5905	35·5890
12	37·6050	37·6050	37·6031

TABLE 4

Comparison between the present finite element results and calculations based on [2], *first mode*, $R = 1$, $\gamma = 0$

	$\bar{\alpha}$		
λ	Present, 10 elements	$M = 10$, $N = 3$ [2]	$M = 1$, $N = 15$ [2] (exact)
0	3·51602	3·51602	3·51602
1	3·88883	3·88883	3·88882
2	4·83369	4·83369	4·83369
3	6·08177	6·08177	6·08175
4	7·47509	7·47509	7·47505
5	8·94046	8·94046	8·94036
6	10·4441	10·4441	10·4439
7	11·9694	11·9694	11·9691
8	13·5079	13·5079	13·5074
9	15·0549	15·0549	15·0541
10	16·6075	16·6075	16·6064
11	18·1642	18·1642	18·1625
12	19·7237	19·7237	19·7215

The results in Tables 1–4 are for $\bar{\alpha} = (\alpha^2 + \lambda^2 \sin^2 \theta)^{1/2}$ and are thus valid for all θ. The definitions of α, λ, R, and θ are the same as in reference [1].

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(*Received* 2 *June* 1980)

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